

Altitude Control of Long-Duration Balloons

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Altitude control enables balloons to perform atmospheric soundings, avoid restricted airspace, and exercise rudimentary navigational control. Reversible altitude control, which does not expend lift gas or ballast, is particularly attractive because long-duration flights can be achieved. Methods of reversible altitude control, including air ballast and mechanical compression, are compared. A new system, described here as differential expansion, is ideally suited to near-neutral density operation and has many advantages over existing altitude control systems including cost, ease of construction, and altitude range.

Nomenclature

F	=	base-to-apex compression force
g	=	gravitational acceleration constant
H	=	atmospheric scale height.
m	=	total balloon mass including payload
P_0	=	atmospheric pressure at $Z = 0$ (e.g., sea level)
R	=	mass-based gas constant ($287 \text{ J Kg}^{-1} \text{ K}^{-1}$)
r	=	equatorial radius of balloon
T	=	ambient temperature
W	=	energy consumption per unit altitude change
Z	=	altitude
Z_{\max}	=	maximum balloon operating altitude
Z_{\min}	=	minimum balloon operating altitude
α	=	balloon diurnal thermal cycle constant
ΔP	=	balloon superpressure

Background

EFFICIENT altitude control is a major challenge for balloon designers. Although traditional zero-pressure balloons are approximately metastable and can, in theory, be driven up or down with minimal buoyancy changes, in practice, solar heating, infrared cooling, and atmospheric turbulence upset this delicate equilibrium. Substantial effort must therefore be expended to achieve a desired altitude.

The oldest method of reversible altitude control (devised by Jean-Francois Pilatre de Rozier in 1785) involves changing the temperature of the lift gas. This category of altitude control includes common hot-air balloons as well as dirigibles and blimps that use engine exhaust to heat their ballonets. Because heat loss across the balloon envelope is substantial, thermal altitude control is inefficient from an energy perspective. It is therefore not considered further in our analysis.

Several other methods of altitude control are also not considered here. These include conventional ballast and bleed systems, which rapidly expend their extra ballast and lift gas, cryogenic systems, which rely on a finite supply of liquefied lift gas, and propulsion systems, which must run nearly continuously and therefore consume large amounts of energy. Finally, several systems are not considered because of they are designed to operate at specific altitudes or latitudes, or are not fully controllable throughout the lower atmosphere [e.g., Stratosail (Global Aerospace Corporation), Solar Mongolfieres,¹ and reversible fluid systems].

This paper examines and compares three systems for altitude control that 1) are highly efficient, 2) have positive and predictable altitude control under most conditions (excluding precipitation), 3) are capable of traversing a substantial portion of the atmosphere, and 4) can operate on long-duration flights of weeks to months.

The most successful and widely known system to meet these criteria is air ballast.^{2,3} Air-ballast (AB) systems make use of the high molecular weight of air relative to the lift gas; to descend, ambient air is pumped into a bladder inside a superpressure balloon (Fig. 1, left). Because the superpressure balloon does not expand appreciably during this process, the density of the balloon increases with time. To ascend, the process is reversed, and air is released from the internal bladder into the surroundings.

AB balloons have evolved rapidly since the early 1990s. Particularly well suited to following air masses in the lower atmosphere, these balloons have become increasingly sophisticated in both their construction and tracking ability. AB balloons have been successfully deployed on a number of major research campaigns^{4,5} and have been considered as possible platforms for new weather balloons.⁶

The second altitude control system that we analyze is based on mechanical compression (MC) of a pumpkin-type superpressure balloon (Fig. 1, center, and Fig. 2). The pumpkin balloon is ideally suited to compression because its structure provides two natural nodes as attachment points for a winch (i.e., the base and apex fittings). By drawing these nodes together, the balloon's volume can be reduced by as much as 50%. This volume change increases the effective density of the balloon, causing it to descend. Although there are many practical problems with implementing MC, it has appreciable advantages over AB including higher efficiency (i.e., less energy is required for a given altitude change) and the potential to operate at high altitudes where pumps and blowers become less effective.

Against this backdrop, we develop a new altitude control system described here as a differential expansion (DE). In a DE system (Fig. 1, right), a lift gas is transferred between a superpressure vessel and a zero-pressure envelope. To increase altitude, lift gas is released from the superpressure vessel into the zero-pressure envelope. To descend, lift gas is pumped from the zero-pressure envelope into the superpressure vessel. By sizing the superpressure vessel and

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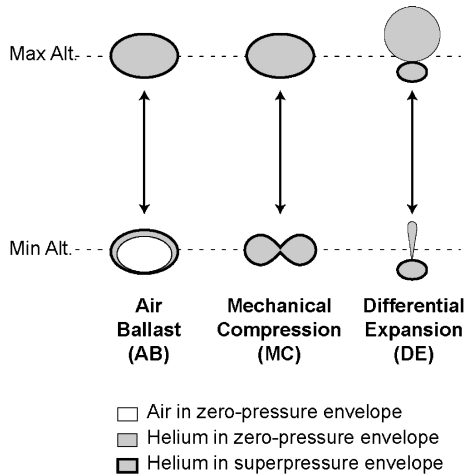


Fig. 1 Schematic diagrams of the three altitude control systems analyzed in this paper.

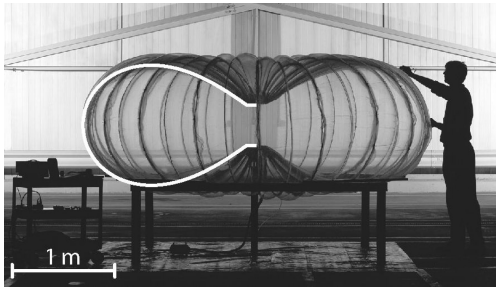


Fig. 2 Compressed pumpkin balloon with the modeled shape superimposed (white line).

zero-pressure envelope according to the relationships presented in this paper, altitude ranges in excess of 20 km and efficiencies exceeding that of MC can be achieved. Furthermore, for a given payload DE systems are simpler to construct and far lighter than either AB or MC systems. Finally, DE systems can be configured with the superpressure vessel inside the zero-pressure envelope so that any lift gas that leaks from the high-pressure system is recovered. This configuration is ideal for long-duration flights.

Air Ballast

Although unquestionably successful in practice, air ballast (AB) systems have several shortcomings that provide opportunities for improvement. Most notable is the “parasitic mass” added by the air ballast itself; as air is pumped into the internal bladder during descent, the balloon becomes progressively heavier. To descend by one scale height, for example, the total mass of the balloon must more than double. With decreasing altitude, AB balloons become increasingly massive and therefore consume proportionally more pump energy for altitude changes and vertical accelerations.

AB systems are also compromised by the large superpressure balloon required to contain the lift gas at the high end of their altitude range. The superpressure balloon adds substantial weight because of its large size and the heavier construction required to compensate for the skin stresses as a result of its large radius of curvature. Strain, creep, and leakage of lift gas become more acute in large superpressure balloons, presenting a particular challenge for high-altitude AB systems.

AB systems also suffer from problems related to their “open” design. Ambient air pumped into the internal bladder invariably contains some moisture. Under unfavorable conditions, this water can condense, or worse, freeze inside the balloon, leading to tears in the fragile air bladder and loss of lift gas from the system. Because of these challenges, AB balloons have been most successful at altitudes lower than approximately 5 km.

A hybrid air-ballast and zero-pressure (ABZP) system (e.g., Earthwinds balloon) is, in theory, a highly efficient means of controlling altitude; our analysis shows that the efficiency an ABZP system can even exceed that of MC system in the limit of an infinitesimally small air-ballast balloon (entire lift provided by the zero-pressure envelope). The tradeoff for this high efficiency, however, is an unrealistically high superpressure. ABZP systems, like AB systems, are also vulnerable to internal ice formation and carry excessive parasitic mass at lower altitudes.

Mechanical Compression

One of the more promising recent developments, MC altitude control overcomes some of the shortcomings of AB systems. In MC systems, a superpressure balloon is compressed by means of a winch or other constriction device. Proposed and tested by Solomon Andrews in the 1800s (Ref. 7), MC has now become a real possibility because of the availability of strong synthetic materials and the development of the pumpkin balloon structure.⁸ Our research group began developing a mechanical compression pumpkin balloon in 2001, and Yajima⁹ independently developed this same concept and has filed for U.S. and Japanese patents on the concept.

MC balloons have an advantage over AB balloons at high altitudes because they do not require a pump or blower. They can also achieve very high efficiencies. As shown later in this paper, for a given payload and altitude range MC balloons require much less energy than their AB counterparts.

This section describes the development of a laboratory MC pumpkin balloon and highlights some of the challenges of implementing this form of altitude control. The pumpkin balloon is unique in that forces resulting from its internal pressure are borne by strong meridional tendons; the lightweight envelope, typically polyethylene, bulges out between the tendons with a small radius of curvature that greatly increases its effective strength. In the preferred MC scheme, the base and the apex of the pumpkin balloon are drawn together (Fig. 2), resulting in a toroid-like form that has a substantially smaller volume than the original pumpkin shape. Because the total mass of the balloon and its payload are conserved, this change in volume results in a change in the effective density and therefore the altitude of the balloon.

Two features of balloons, and in particular, pumpkin balloons, make mechanical compression feasible. First, the tension on the meridional tendons, which is proportional to the compression force, remains approximately constant as the balloon is compressed because the decreasing radius of curvature counteracts the effect of increasing superpressure. Second, most of the force required to contain the balloon’s superpressure is borne by the tendons, providing a natural form of leverage for the compression winch.

The maximum base-to-apex force [Eq. (1)], which occurs when the balloon is fully compressed (base and apex in contact), is a function of the superpressure and cross-sectional area of the balloon. The square root in the denominator accounts for the fact that the tendons intersect the base and apex at approximately a 45-deg angle.

$$F \approx \pi r^2 \Delta P / (1 + \sqrt{2}) \quad (1)$$

For a pumpkin balloon 10 m in diameter with 15% superpressure at 17 km altitude (9 kPa), the theoretical force required at full compression is approximately 45 kN. In an atmosphere having a scale height of 7 km, full expansion would cause the balloon to ascend by approximately 4.2 km.

Because compression force and energy are both proportional to the superpressure, altitude control becomes virtually free when the superpressure approaches zero (e.g., during the night at all latitudes, in the polar winter, or in continuous sunlight). Under such circumstances, even the largest superpressure balloons could probe the atmosphere with lightweight compressing mechanisms and extremely low power consumption. A particularly efficient navigation scheme, essentially control and hold, would change the balloon’s altitude only during the night, shutting off the compression mechanism and holding the sunrise altitude throughout the daylight hours.

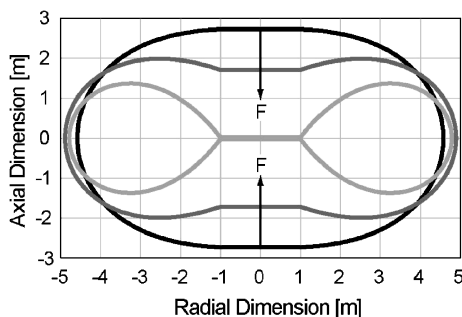


Fig. 3 Model of a MC pumpkin balloon having large base and apex fittings for maximum compression. Full compression requires a force of 45 kN and reduces the balloon volume by approximately 45%.

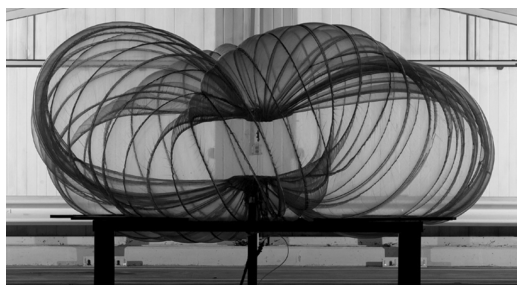


Fig. 4 Natural contortion of a pumpkin balloon during compression. Photo credit: Thom Kendall.

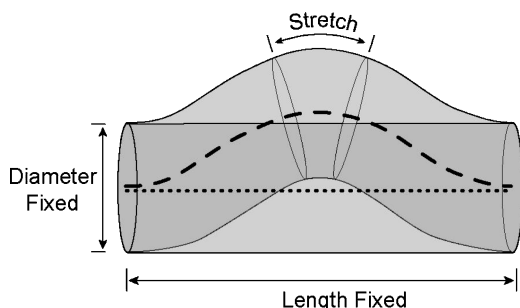


Fig. 5 Conceptual model shows the cause of the contortion in a compressed pumpkin balloons (see text).

We develop a numerical model of a base-to-apex compression using the “shooting” method¹⁰ to calculate the balloon shape and volume as a function of the compression force and superpressure (Fig. 3). The gore shape is then calculated in order to achieve a constant local radius of curvature and therefore a uniform circumferential stress field in the envelope of the uncompressed balloon.

Model results were compared with laboratory measurements on an actual 3-m diam balloon (Fig. 2, white line). The modeled and observed dimensions, internal pressure (50 Pa), and compression force (165 N) agree to within approximately 10%.

The compressed volume, which is ideally as small as possible, can be further decreased by increasing the size of the base and apex fittings or rings (as in Fig. 3) and by using circumferential tendons to prevent bulging at the balloon’s equator. With these methods, it is theoretically possible to compress a pumpkin-type balloon to less than 50% of its original volume.

The compressed pumpkin balloon shown in Figs. 2 and 3 is, however, structurally unstable (Fig. 4); it will assume a distorted shape that is similar to, but more extreme, than that observed in larger conventional pumpkin balloons.¹¹ To demonstrate why this distortion occurs, consider a fully compressed pumpkin balloon that is “unrolled” to form a tube of fixed overall length and diameter (Fig. 5).¹² Although this imaginary tube is confined at its ends, it can elongate by assuming a sinuous shape. The sinuous shape has

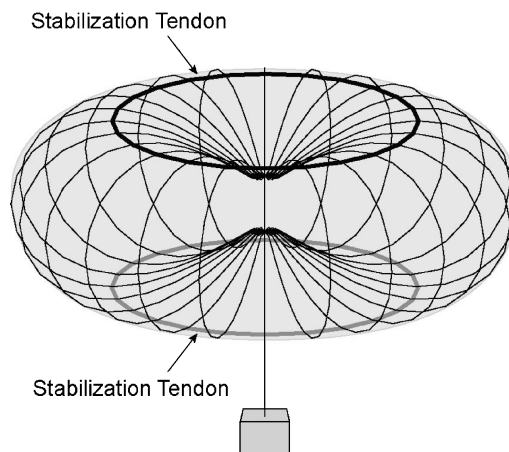


Fig. 6 Pumpkin balloon design with two circumferential stabilization tendons that does not become contorted during compression.

a larger volume (and lower internal pressure) making it more stable than the properly deployed balloon.

One possible solution to the deployment problem employs circumferential stabilization tendons that encircle the balloon approximately midway between the equator and the poles (Fig. 6). The stabilization tendons are attached to each meridional tendon, directly preventing the elongation inherent in the distorted shape. This solution is consistent with earlier findings that minimizing excess envelope material near the poles is critical to achieving proper deployment of fixed-volume pumpkin balloons.¹¹ Our laboratory tests show that stabilization tendons are even more effective than removal of excess envelope material because the tendons are strong, inelastic, and apply their stabilizing force precisely where it is required. The conceptual model in Fig. 5 suggests that circumferential tendons (or strong envelope material at the same location) could also improve the deployment of constant-volume ultra-long-duration balloons.

Actual implementation of MC altitude control is challenging because of the magnitude of the forces involved. For the small 10-m balloon modeled in Fig. 3, a high-efficiency winch having a total mass of approximately 1.5 kg must be capable of generating a force of up to 45 kN over an extent greater than 5 m. In laboratory tests, our custom traction winch coupled to a distributed block-and-tackle system shows some promise in meeting these specifications. The distributed pulleys minimize strain on the balloon’s base and apex fittings. Furthermore, higher compression forces can be achieved simply by adding more pulleys. Because the pulleys can also be attached directly to the balloon’s tendons, other more radical compression schemes, including circumferential constriction, can be tested experimentally.

Despite its apparent feasibility and the promise of high efficiency, MC altitude control is likely to be complex and expensive in practice. As with AB systems, the superpressure balloon must be large enough to accommodate all of the lift gas at the highest altitude and strong enough to contain the specified fractional superpressure at the lowest altitude. Probably the most significant obstacle, however, is its limited altitude range, as full compression can reduce the volume of a pumpkin balloon only by half, corresponding to a few kilometers of vertical travel.

Differential Expansion

Differential expansion, in comparison with MC, is simple and inexpensive. A DE system consists only of a superpressure vessel (usually a reinforced balloon) and zero-pressure envelope connected via a gas transfer device (e.g., a pump and valve) that carefully regulates the distribution of the lift gas. In the preferred design, the superpressure vessel and gas transfer device are placed inside the zero-pressure envelope so that any lift gas that leaks from the high-pressure system is recycled (Fig. 7). To further conserve lift gas, the zero-pressure envelope can be completely sealed so long as it is large enough to accommodate the lift gas at the balloon’s maximum operating altitude.

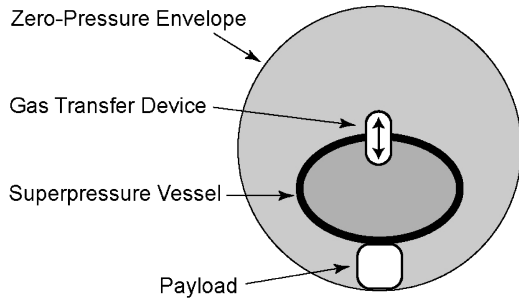


Fig. 7 DE balloon.



Fig. 8 DE balloon in flight over Western Massachusetts. The upper (white) balloon is released to terminate the flight and is not part of the DE system.

In general, the superpressure vessel is made as large as possible in order to support the maximum deviation from neutral buoyancy. The superpressure vessel is therefore sized to lift the entire mass balloon mass (payload, envelopes, and hardware) at the lowest operating altitude. To make the balloon ascend, lift gas is released from the superpressure vessel into the zero-pressure envelope, whereas to descend the flow is reversed. The maximum operating altitude is determined by the volume of the zero-pressure envelope. For acceptable control and efficiency at sunrise, however, the volume of the zero-pressure envelope should not exceed that of the superpressure vessel by more than a factor of 20 or so. This constraint limits the altitude range (maximum minus minimum altitude) to approximately 20 km in the Earth's atmosphere.

In calm conditions and in the absence of diurnal heating and cooling, the volume of the superpressure vessel is not as critical. Functional laboratory prototypes have been constructed using a trash bag for the zero-pressure envelope and a 2-liter soda bottle for the superpressure vessel. This bag-and-bottle system can be considered to be operating at the upper end of its altitude range as the minimum operating altitude is tens of kilometers below sea level.

A larger DE balloon has been constructed using a 150-liter superpressure vessel inside zero-pressure envelope made from commercial-grade aluminized nylon. Although not optimized for control, the superpressure vessel is large enough to counter the effects of solar heating in the zero-pressure envelope. This DE balloon is currently undergoing testing in the atmosphere and has been flown to altitudes of 1.6 km above sea level (Fig. 8).

Model Comparison

A simple model has been developed for systematically comparing AB, MC, and DE balloons. To allow the derivation of explicit

expressions for superpressure and energy consumption, we base our analysis on an isothermal atmosphere and constant scale height. The mass the balloon includes the payload, the envelopes, and helium. The diurnal thermal cycle constant gives the fractional change in buoyancy caused by solar heating and infrared cooling of the lift gas (typically 0.05–0.15).

$$\Delta P_{DE} = \alpha P_0 e^{-Z_{Min}/H} \quad (2)$$

$$\Delta P_{AB} = \alpha P_0 e^{-Z_{Min}/H} \quad (3)$$

$$\Delta P_{MC} = \alpha P_0 e^{-Z/H} \quad (4)$$

$$W_{DE} = m g e^{(Z_{Min} - Z)/H} \ln(1 + \alpha e^{(Z - Z_{Min})/H}) \quad (5)$$

$$W_{AB} = m g e^{(Z_{Max} - Z)/H} \ln(1 + \alpha e^{(Z - Z_{Min})/H}) \quad (6)$$

$$W_{MC} = m g \alpha \quad (7)$$

Equations (2–4) are plotted in Fig. 9 (balloon superpressure vs altitude). Equations (5–7) are plotted in Fig. 10 (theoretical work

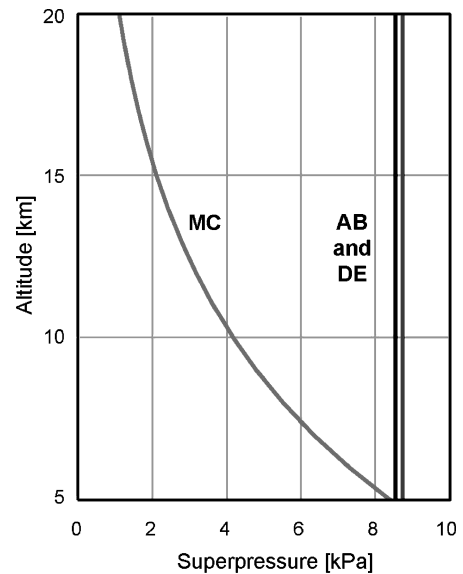


Fig. 9 Superpressure vs altitude for AB, MC, and DE balloons.

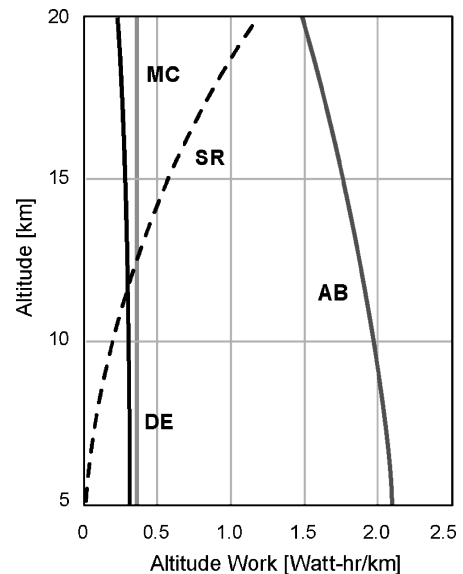


Fig. 10 Work required per kilometer of descent (1 kg total balloon mass) for AB, MC, and DE balloons: sunrise work for the DE balloon in units of Watt-hours.

required per kilometer of altitude change vs altitude). In this comparison $m = 1$ kg, $\alpha = 0.15$, $T \sim 300$ K, $P_0 = 101.3$ kPa, and the altitude range is as shown.

The superpressure required for DE and AB systems is constant with altitude while MC superpressure drops exponentially during ascent. The maximum superpressure for all three systems, however, is identical. The lower superpressure of the MC system therefore confers no structural or weight-saving advantage because it must be designed to contain the same maximum pressure as the DE and AB systems. In effect, the MC system has wasted capacity above its minimum altitude as it is structurally capable of holding more lift gas. The DE system makes use of this excess capacity to counter the diurnal thermal cycle in its zero-pressure envelope.

The work required per kilometer altitude change reveals the inefficiency of conventional AB altitude control. At low altitudes, AB suffers from the parasitic mass of the ballast air whereas at high altitudes it has high energy consumption as a result of excessive superpressure (in comparison with MC) and excessive superpressure volume (in comparison with DE). These differences in efficiency and superpressure become progressively larger with increasing altitude range ($Z_{\max} - Z_{\min}$). As the altitude range approaches zero, all three systems converge to the same performance.

For DE balloons, the maximum energy consumption occurs at the minimum operating altitude; as the DE balloon ascends, it increasingly takes on the character of a zero-pressure balloon, and its energy consumption decreases toward zero. Thus, higher efficiencies can always be achieved by decreasing the minimum altitude (decreasing the volume of the superpressure vessel). As the superpressure vessel becomes smaller, however, it must be designed to withstand greater superpressures, and the DE balloon becomes more difficult to control.

Although DE systems are preferable from the perspective of efficiency and simplicity of construction, they do require that lift gas be pumped into the superpressure vessel at sunrise. This sunrise work (dashed line in Fig. 10) is not required for AB or MC balloons. Sunrise work increases with altitude but is generally modest in comparison with the energy required for vertical motion. DE systems also must operate closer to neutral density than either MC or AB systems because they have a smaller superpressure volume. DE altitude control therefore might not be well suited for rapid ascents, for offsetting changes in payload mass (e.g., unloading cargo from an airship), or for countering precipitation loading. Finally, DE systems will generally require positive displacement compression; blowers, turbines, and other aerodynamic pumps would have to operate at exceedingly high speeds, or be very large, in order to compress the low-density lift gas.

These results are expected to be robust for relatively small vertical motions where the atmosphere is approximately isothermal. For larger excursions, a numerical model with a fixed lapse rate gives similar results with the exception that the efficiency advantages of the DE balloons become more pronounced.

Discussion

Based on the preceding analysis and descriptions, DE balloons appear to have some significant advantages over AB and MC balloons. For a given gross mass and altitude range, a DE balloon will have the following: 1) a substantially smaller superpressure volume that can be constructed from lighter materials because of its smaller radius of curvature; 2) lower energy consumption per unit altitude change caused by the efficiency of the altitude control; and 3) lower gross mass [as a result of 1) and 2)] that decreases energy consumption to below that shown in the conservative comparison in Fig. 10.

Furthermore, because DE systems are closed they are not prone to the icing problems of AB systems. And because the superpressure

vessel can be placed within the zero-pressure envelope, the loss of lift gas to the environment is slowed dramatically. DE systems are ideally suited to long-duration flights.

The weight and energy benefits just described not only reduce the cost of the DE balloon, but also improve its performance. Small and light DE balloons are easily transported and launched. Their internal temperature equilibrates rapidly with their surroundings, enabling them to easily penetrate thermal inversions. Small DE balloons may also be safer for aviation and overflights, allowing them to be used more widely in the atmosphere.

Conclusions

We analyze and compare several methods of balloon altitude control and develop a new system described as differential expansion. Differential expansion is shown to have some substantial advantages over air ballast and mechanical compression altitude control, especially when operating at near neutral density. These advantages include significantly lower cost and weight, greater altitude range, and longer-duration flights than is possible with other systems.

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